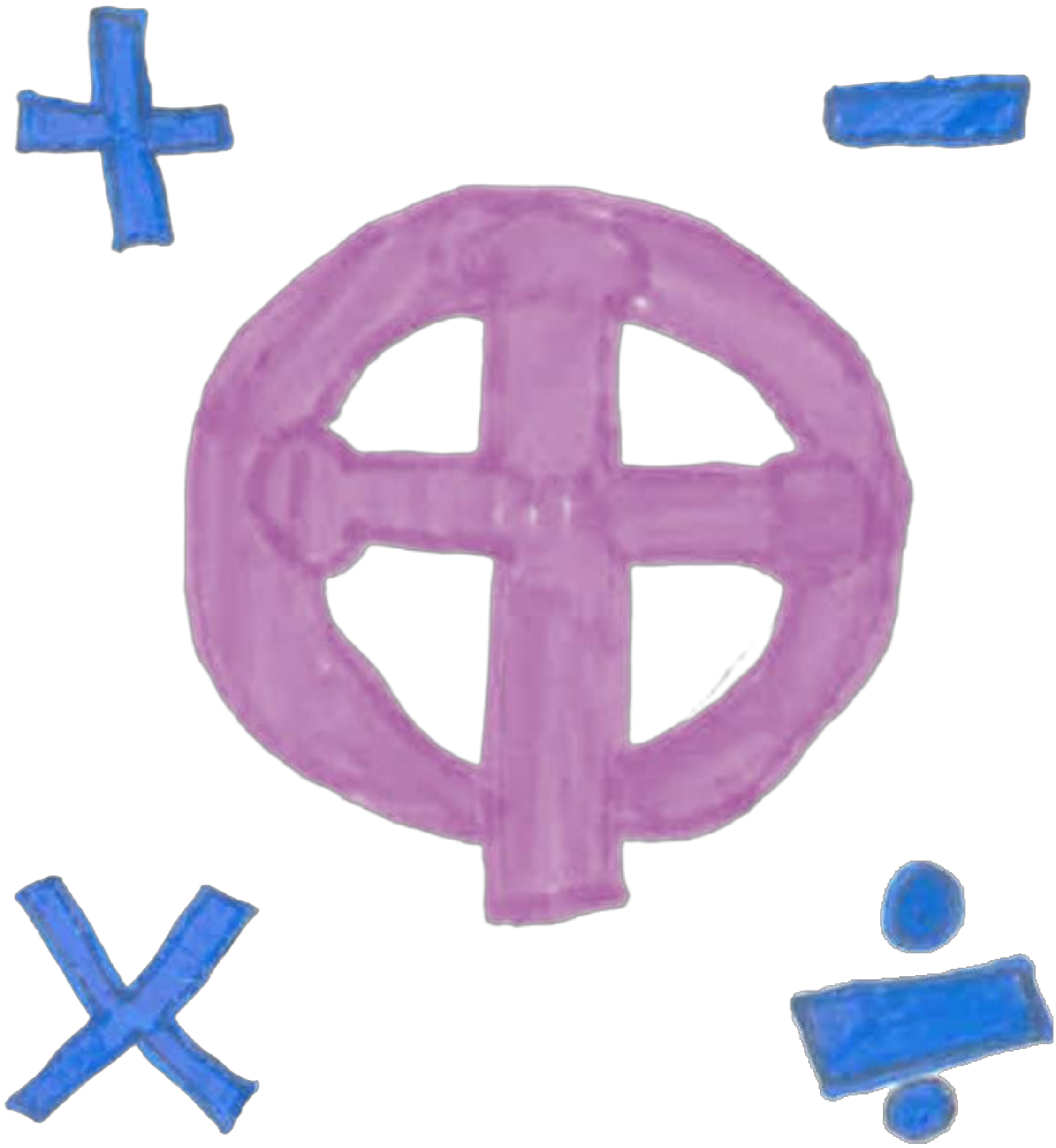

A Reasoning-led Approach to Teaching Fractions

Mathematical Sciences into Schools: Project Report

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Abstract

This project was delivered at [redacted] Primary School, focusing on supporting Year 6 pupils who require targeted support with fractions. Fractions were chosen as pupils were struggling to identify and apply the correct methods across the core operations of addition, subtraction, multiplication, and division. The pupils' weak procedural recall was caused by their fragmented understanding, so a reasoning-led approach was taken, aiming to develop their conceptual understanding of why these methods work. Additionally, the project aimed to support pupils in distinguishing between the different methods, and in improving their confidence and independence with fractions.

Equivalence, lowest common multiples, and simplest form were included alongside the core operations as supporting ideas to help underpin the methods conceptually. A scaffolded cycle was used throughout the worksheets, informed by the Zone of Proximal Development. This cycle guided the pupils towards greater understanding via temporary teacher support, which was gradually removed to build independence. Discussion and writing down of methods helped to diagnose misconceptions and refine reasoning. The project was constantly adapted, both during sessions and between them, to adjust for the pupils' engagement and progress with the content.

Overall, the project was largely successful in supporting conceptual discussion, confidence, and independence within the sessions. However, the successes, conclusions, and legacy of the project are limited by its small scale, timetable disruptions, and reliance on responsive delivery.

1. Introduction

1.1 Context

This project was carried out in partnership with [redacted] Primary School. In a meeting discussing their approach to intervention and pupil progress, a maths advisor from the school's trust defined the term "spotlight pupil" as any pupil who is not on track in mathematics and therefore requires support via one-to-one interventions. These interventions are led throughout all year groups to identify and eliminate pupils' gaps and misconceptions.

The project took place over six weeks, led by a mathematics undergraduate from Durham University. Two 45-minute slots were allocated each Wednesday afternoon, with one pair of Year 6 spotlight pupils attending each session; the same pairs were maintained across the project. Since the project took place during curriculum time, it was important to ensure that the content was chosen with the Year 6 teacher's priorities in mind. The teacher identified fractions as an area for targeted improvement after spotlight pupils had dropped marks in that area on a recent mock SATs paper. She hypothesised that this was a result of pupils confusing similar methods across the core fractions operations: addition, subtraction, multiplication, and division. Seigler and Pyke (2013) corroborate this, finding that students frequently confuse fraction operations, inappropriately using elements from other fraction methods in 55% of answers to division problems and 46% of answers to multiplication problems.

The Year 6 teacher's idea was that the pupils' fragmented fractions learning across KS2 had led to fragmentation in their understanding. One-to-one SATs interventions confirmed this issue when Year 6 spotlight pupils were working through numeracy papers. They struggled to begin with fractions problems, suggesting that they were unsure of what method to apply. After some prompting, they would remember fragments of different methods, but would never apply the correct full procedure, evidencing the teacher's concerns.

1.2 Rationale

An anecdote from a SATs one-to-one intervention provides clear motivation for the aims of the project. When attempting a problem involving multiplication of two decimal numbers, one spotlight pupil did not apply column multiplication, as he did not associate that method with the question. Instead, he attempted the problem using his own mathematical reasoning. After reaching an incorrect solution, he was questioned on each step of his method until he discovered his mistake. Upon correcting this error, further discussion led him to the realisation that his method was effectively column multiplication written out horizontally. His conceptual breakthrough then enabled him to more confidently apply column multiplication in the following problems. This indicated that some spotlight pupils respond well when discussion is used to make the reasoning behind a method explicit, as this supported confident and precise application of the method itself. Reasoning mathematically and correcting mistakes is what led to increased fluency in applying methods, helping to motivate this project's focus of teaching with a reasoning-led approach via discussion. This aligns with guidance on teaching for mastery from the National Centre for Excellence in the Teaching of Mathematics (NCTEM), which states that "procedural fluency and conceptual understanding [should be] developed in tandem because each supports the development of the other" (NCTEM, 2022).

1.3 Aims

Consequently, the project had four main aims:

- to lead teaching with the reasoning behind why fraction methods work;
- to support pupils in distinguishing between the methods used for the core fraction operations;
- to encourage pupils to refine their conceptual understanding of fractions through discussion; and
- to support pupils in developing greater confidence and independence in fractions.

2. Mathematical Content and Rationale

The mathematical scope of the project was narrowed from the broader 'KS2 Fractions' to a selected set of a mathematical aims, for pupils to be able to:

- "add and subtract fractions with different denominators using the concept of equivalent fractions";
- "multiply simple pairs of proper fractions, writing the answer in its simplest form";
- "divide proper fractions by whole numbers"; and
- "associate a fraction with division".

These aims were chosen to align with the Year 6 National Curriculum (Department for Education, 2013, p. 40). These topics were all within the scope of the project because they are directly associated with its focus in securing understanding of the core operations. Finding equivalent fractions, lowest common multiples (LCM), and putting fractions in their simplest terms were incorporated into the project as they are necessary to explain and carry out these operations. Introducing these topics as supporting ideas that underpin the methods, rather than as isolated topics, was important in enabling a reasoning-led approach.

Other curriculum-relevant topics, such as improper fractions and mixed numbers, were excluded from the project to prioritise conceptual depth within the available time. Similarly, comparing and ordering fractions was excluded, as this would have broadened the project's focus beyond its main aims. Division of two fractions is not detailed on the curriculum so was also excluded from the project's content. This was to avoid any potential confusion for the spotlight pupils before their SATs, despite its potential to be useful for associating fractions with division.

3. Design of the Project

3.1 Lesson Cycle

The Zone of Proximal Development is “the distance between the actual developmental level as determined by independent problem solving, and the level of potential development as determined through problem solving under adult guidance” (Vygotsky, 1978, p. 86). It is a useful educational idea because tasks pitched in a pupil’s ZPD can be completed with a small amount of temporary support. As pupils gain confidence and competence, this guidance can be slowly removed, leaving them with greater independence. This temporary support is often defined as ‘scaffolding’. Wood, Bruner and Ross (1976, p. 90) explain scaffolding as a process that enables a child to solve a problem beyond their unassisted efforts by an adult controlling the elements that are beyond the learner’s capacity; this allows them to concentrate upon the elements that are within their range of competence. Each session in the project was accompanied by a worksheet that was designed around a scaffolded cycle led by one teacher. An example of this can be found in Appendix 1 [A1].

The worksheet cycles begin by pitching the pupils a problem that lies in their ZPD for them to complete independently [A1.1]. The spotlight pupils had worked on fractions throughout KS2 but couldn’t fully recall methods, as identified in one-to-one interventions. Their familiarity with fractions was obscured by their confusion between the methods, thus fractions problems involving the core operations were in their ZPD.

Next, the pupils would write an explanation of how they attempted the problem [A1.2], challenging them to consider the reasoning behind their method. This allows the session teacher to visualise the pupils’ thinking and diagnose any misconceptions. Following this, the teacher demonstrates the correct solution to the problem [A1.3], discussing any differences from the students’ attempted solutions. The intended result of this discussion was for the students to realise their mistakes and then correct or expand upon their written method [A1.4].

Following this, the pupils use their written corrections and/or amendments to attempt practice questions [A1.5]. These questions were intended for consolidation, helping to link conceptual understanding behind methods to their implementation in SATs problems. Teacher support would be removed as the pupils attempted the questions, encouraging them in their gradual move towards greater independence. Their improved written method would act as a smaller scaffold.

The final question, however, would be a challenge designed to break their method [A1.6]. Pupils are prompted to write down what they have identified as different about the challenge problem [A1.7], which gives them a platform to begin considering why the method was unsuccessful. Their responses inform the next discussion [A1.8], allowing the cycle to respond to the pupils' developing ZPD. In this example, the next discussion introduces "equivalent fractions". This was done with the intention of helping pupils to connect equivalence to the method for adding fractions, rather than treating it as a disconnected, standalone topic.

3.2 Sequencing of Topics Across Sections

On a larger scale, scaffolding was used to cover topics at the correct pace across the sessions, with each new topic introduced in sequence. The worksheets were made each week, so each design was informed by the previous week's session; this will be discussed further in *4. Delivery and Adaptations*.

The resources from Session 1 primarily covered addition of two fractions and introduced topics such as equivalence and lowest common multiples.

Each subsequent session was then designed around the following structure:

- recap the operations from the previous sessions;
- introduce the new operation(s), alongside any necessary supporting ideas;
- mixed questions on everything covered so far.

This sequencing was informed by Cognitive Load Theory (CLT), which was designed to assist in optimising intellectual performance by considering how to make best use of pupils' limited capacity working memory. Cognitive load may be affected either by the intrinsic nature of the material (intrinsic cognitive load) or alternatively, by the way the material is presented, or the activities required of students (extraneous cognitive load) (Sweller et al., 1998, p. 251, 259, and 263). Since pupils were already occupied by the intrinsic load of each fraction method, introducing one new operation at a time, and constantly recapping prior knowledge, was intended to reduce any extraneous cognitive demand caused by confusion between each method. The aim was for pupils to develop security in the distinctions between these methods.

3.3 Supporting Materials

As well as the design of bespoke project resources in the form of the worksheets, additional materials were borrowed from online teaching resource banks. The first of which [Appendix 2] was a multiplication grid. Throughout primary school, the spotlight pupils had developed gaps in their mathematical learning, and a common struggle with times tables recall was observed in Year 6 interventions. Multiplication is vital for the core fractions operations, especially when finding a common denominator, so weak times tables recall would overload working memory when developing fractions knowledge. This gap in learning was already being addressed in one-to-one intervention sessions, thus strengthening the pupils' recall was not within the scope of this project. The multiplication grid was included to help mitigate this cognitive load, as the students would have the results for any products they needed to compute. However, reading these results from the grid still requires that the pupils know which factors need to be multiplied, so conceptual understanding was still being developed. This grid in particular was chosen due to the use of colour to separate the times tables from each other. This could help the pupils when finding the LCM as it forces them to read the table in the same direction each time, and thus more easily compare different times tables. This is another way to reduce their extraneous cognitive load.

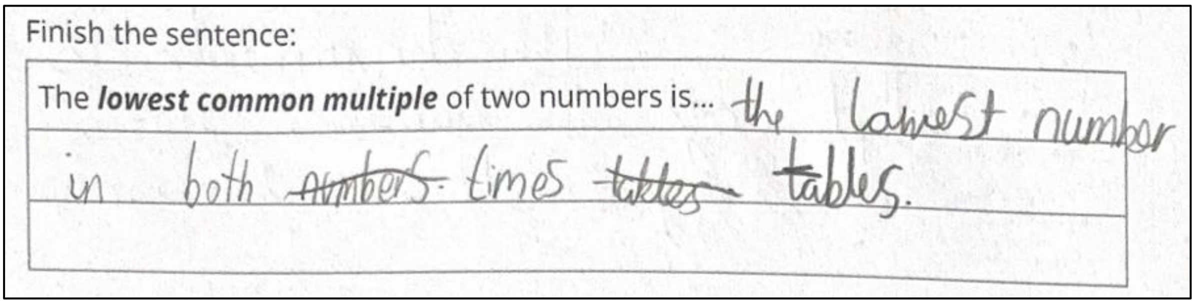
The second external resource [Appendix 3] was a fractions wall. Since the project was designed to develop the pupils' understanding of why the methods work, a fractions wall was useful as a visual reference point against which solutions could be checked. Through this resource, pupils could determine whether the result from their method was plausible and compare it to alternative methods. My decision to include this resource aligns with research suggesting that visual representations help students understand the concepts and procedures of problems with fractions, and facilitate discussion with their peers (Barbosa, 2021, p. 3).

4. Delivery and Adaptations

4.1 Discussion in Practice

A primary tool of the project's delivery was concept-led discussion, which was enabled by responsive teaching from the mathematics undergraduate. The small-group format allowed for close questioning and immediate feedback, with the pupils giving explanations before contribution from the teacher. This created an environment of mutual exploration, which was documented during the project's Durham University observation. The observation record [Appendix 4] describes the session as an "investigation" and notes a "mix of asking students to describe thoughts and answer questions".

An example of guided explanation via teacher prompting appears in the observation record when discussing the teacher's explanation of LCM. The session teacher asked the pupils for the meaning of "lowest", "common" and "multiple". Then, using the supplementary multiplications grid, the teacher demonstrated how this terminology was directly linked to the method the pupils had used to find the LCM. This helped with consolidation, as the pupils recalled the method the following session when prompted to consider the terminology [Figure 1].


Finish the sentence:

The ***lowest common multiple*** of two numbers is... the lowest number in both ~~numbers~~ times ~~tables~~ tables.

Figure 1 – a pupil's handwritten definition of lowest common multiple

Discussion was present throughout sessions as the cycle detailed in *3.1 Lesson Cycle* was closely followed. Pupils were constantly being scaffolded towards the correct methods via discussion. An example of this beyond the worksheets comes from the fractions wall. Since the project aimed to develop the pupils' understanding of why methods work, it was important that the session teacher's method was not simply presented as an algorithm to be copied. Thus, when pupils reached different answers to the same problem, the teacher led them to use the fractions wall to visually check their conflicting results. This made it easier to identify mistakes in the incorrect method, leading to a group discussion about why the correct method worked.

4.2 Writing in Practice

In practice, the pupil's written work revealed misconceptions that would not have been visible from their numerical answer alone. Throughout the sessions, they wrote methods and explanations alongside their answers, showing whether they had a secure grasp on the reasoning behind each step of the method. An example of this can be seen in Figure 2.

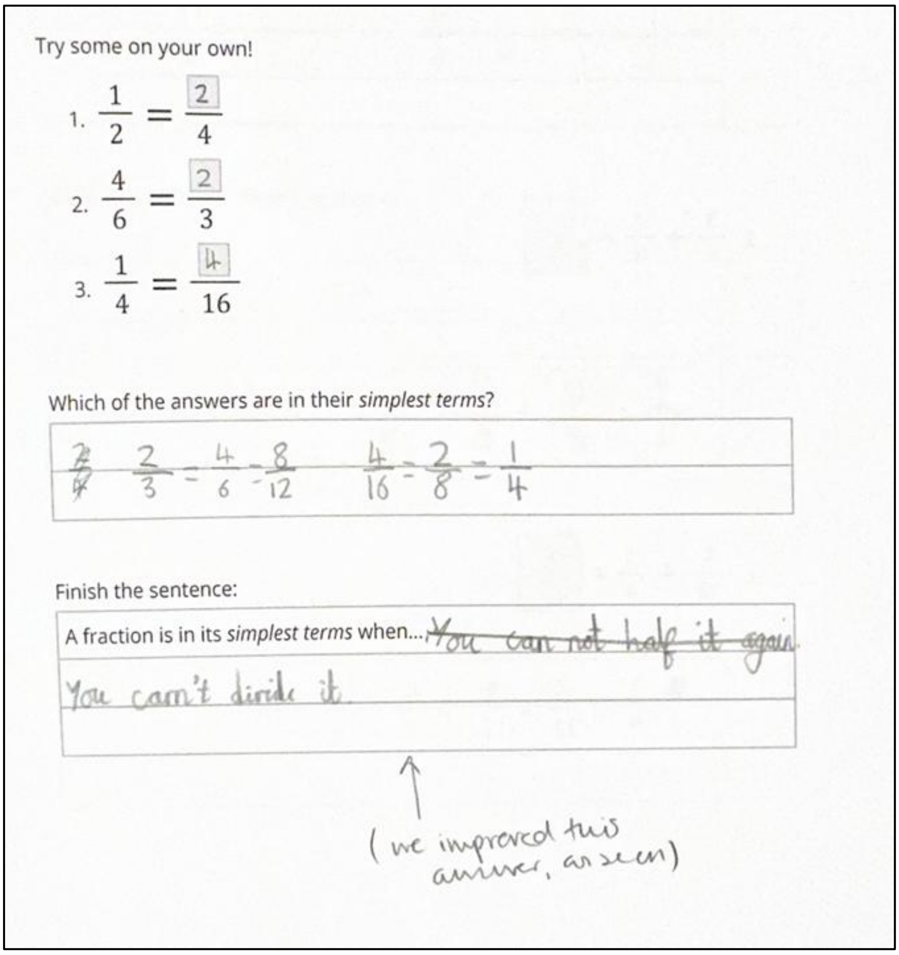


Figure 2 – a pupil's numerical answers to simplest terms problems, alongside their improved, handwritten definition

Here, despite her correct numerical answers, the pupil's definition of "fractions in their simplest terms" was not fully correct; her written answer gave insight into an otherwise hidden misconception. The pair of pupils were prompted to read their definitions to each other and, guided by prompting from the session leader, they came to a more accurate mutual conclusion. As seen in Figure 2, this correction was made by crossing out the previous answer, rather than erasing it. This was encouraged during delivery as it kept the error visible so that it could be discussed alongside the correction. Metcalfe (2017, p. 465) argues that "errorful learning followed by corrective feedback is beneficial to learning" and that "analysis of the reasoning leading up to the mistake is crucial". The pupil's improvement can be seen in her more accurate answer to the same question in the following recap session [Figure 3].

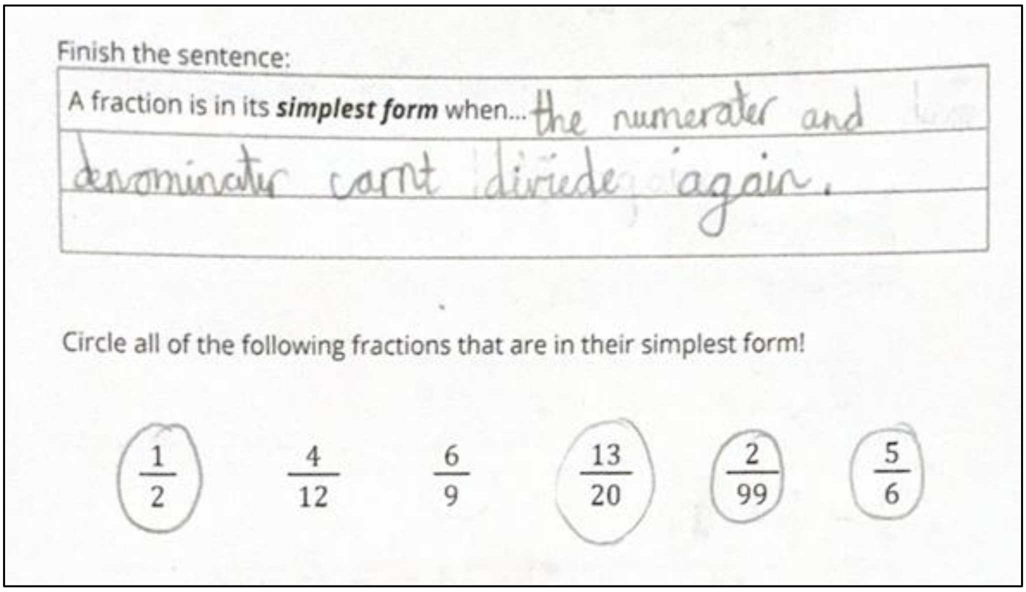


Figure 3 – a pupil's more accurate definition of simplest form

Here, the pupil's answer was improved further via discussion, with the inclusion of "numerator" and "denominator". This was supported by her understanding of the role of division in this definition, which was retained from the previous week. A more rigorous definition would mention that the highest common factor of the numerator and denominator is 1, however this improvement would introduce unnecessary terminology that could obscure the pupils' intuitive understanding of the concept. Allowing the pupils to use their own definitions reduced cognitive load in recall, as the extraneous load of interpreting somebody else's words was removed. A key example of this in delivery was in the final recap session; one pupil's sheet can be found in Appendix 5. The pupils kept these sheets to revise from.

4.3 Adaptations to the Worksheets

In practice, the worksheets were developed week-by-week, with later worksheets being informed by the delivery of earlier sessions. This meant that the content of each session could respond to the topics that pupils were finding difficult. The worksheets were also often adapted in response to pupils' progress during the session. For example, in the first session, an impromptu extension was required when pupils in one of the pairs completed the planned final questions more quickly than expected. These questions were simply handwritten by the teacher during the session.

Another adaptation made to the worksheets was to help maintain pupil focus. With the sessions taking place in the afternoon, pupils were often mentally drained. Their capacity for mathematics was especially depleted as they had already completed their maths lesson in the morning. This sometimes led to a temporary loss of focus, especially in the latter half of the sessions, as the pupils' working memory was more overloaded. Involving pupils in discussion helped to mitigate this, and, for the most part, focus was maintained throughout the sessions. This is evidenced in the observation record, which notes "it was very impressive to keep students so engaged" [Appendix 4].

The method-definition exercises created the extraneous cognitive load of writing, a task that was more difficult for some of the spotlight pupils. To mitigate this, when the session leader noticed a decline in focus, writing tasks were instead made discussion-based. After the first session, future worksheets were adapted to sometimes use fill-in-the-blank-style exercises. This was intended to balance pupil engagement with the pedagogical strength of writing methods, optimising the depth of conceptual understanding that the pupils could gain. Examples of this can be seen in Figure 4 and Figure 5.

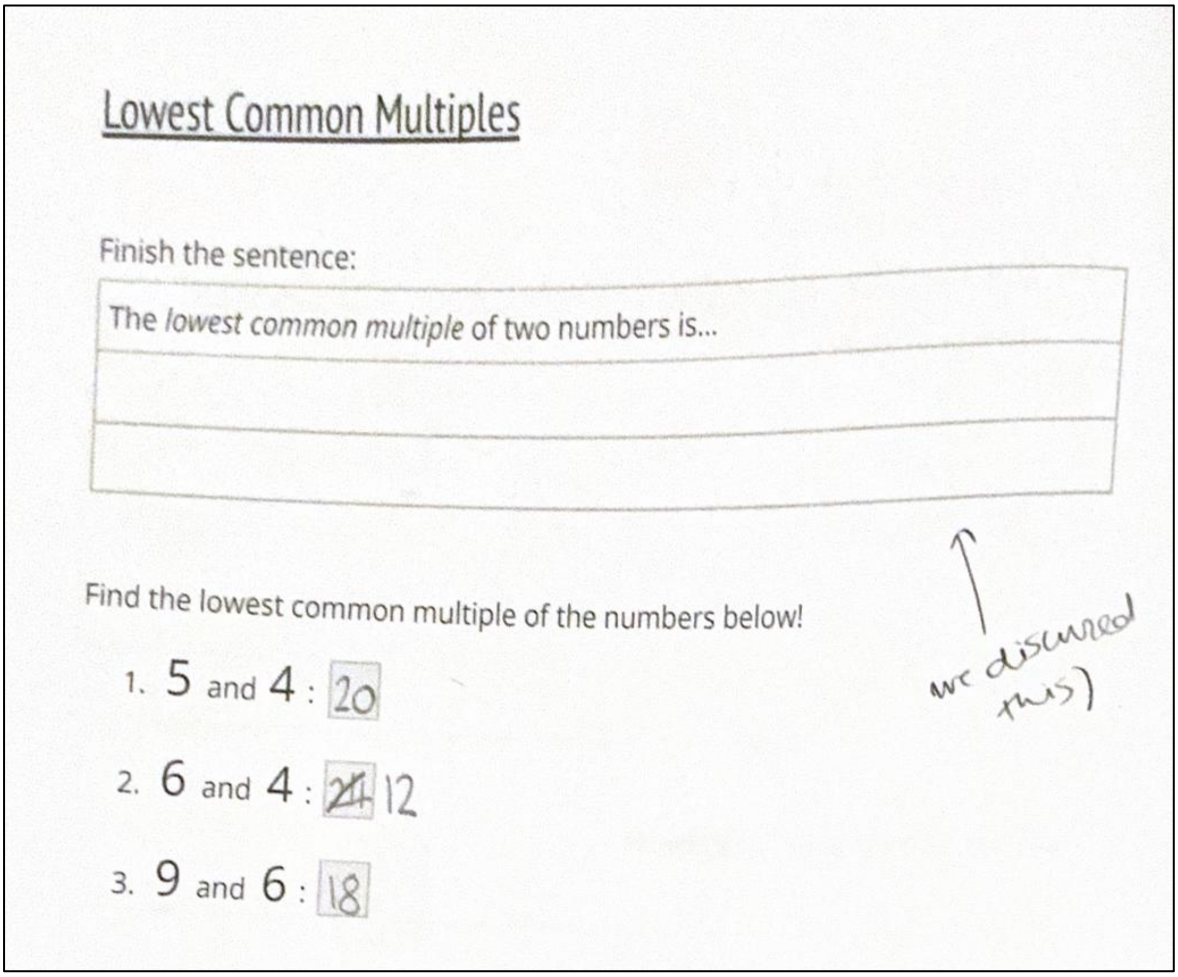
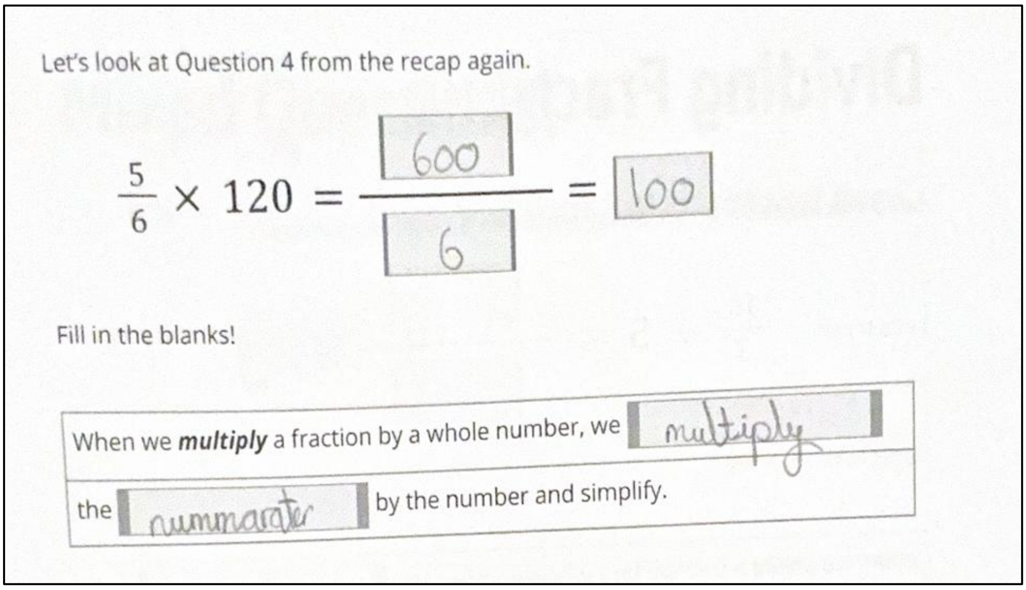


Figure 4 – a box intended for a handwritten answer, instead left for discussion due to a decline in pupil focus


Let's look at Question 4 from the recap again.

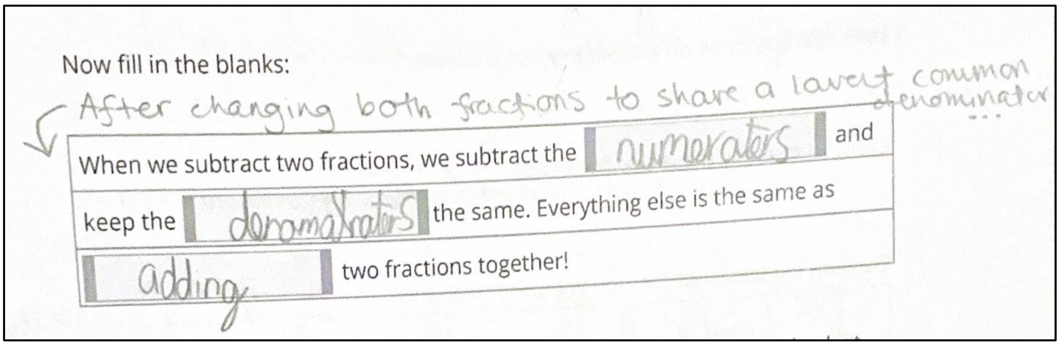
$$\frac{5}{6} \times 120 = \frac{600}{6} = 100$$

Fill in the blanks!

When we ***multiply*** a fraction by a whole number, we multiply the nummarater by the number and simplify.

Figure 5 – examples of fill-in-the-blank-style activities, used to mitigate extraneous cognitive load

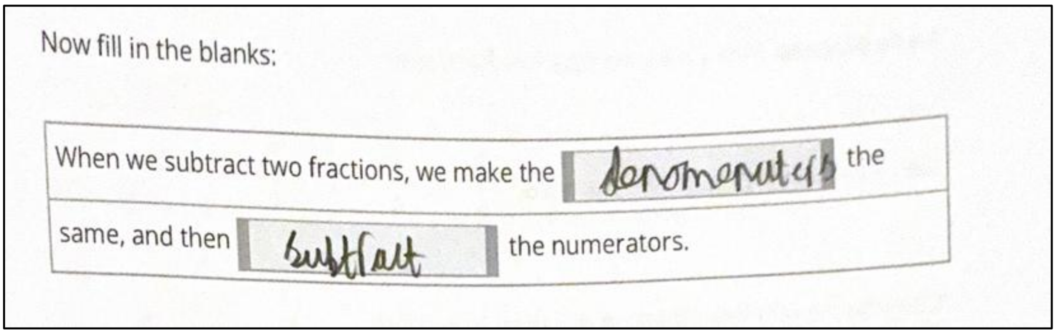
The wording of some worksheets also had to be refined based on pupil response. In the second session, a pupil pointed out a conceptual mistake in a 'fill-in-the-blank' activity, which can be seen in Figure 6. Figure 7 shows the improved wording for the following group. This demonstrated that pupils were engaging with the concepts confidently enough to question the teaching materials. The adaptation prevented any confusion for the other group, but provided a useful teaching moment as the mistake was explained in the following session. This example gave the pupils more exposure to refining mathematical definitions.


Now fill in the blanks:

After changing both fractions to share a lowest common denominator ...

When we subtract two fractions, we subtract the numeraters and keep the denomanaters the same. Everything else is the same as adding two fractions together!

Figure 6 – worksheet error, with handwritten correction from session teacher


Now fill in the blanks:

When we subtract two fractions, we make the denomenaters the same, and then subtract the numerators.

Figure 7 – permanent rewording worksheet error

4.4 Adaptations to the Session Structure

The intended session structure was disrupted by school events and timetable constraints that were outside the control of the project. Six weeks of sessions were originally allocated, but this was not possible in practice. In the second week of the project, the school had a Religious Education inspection, so the two sessions were cancelled. The following week was the half-term break, and the week after that suffered a timetable clash in which one pair could not attend their session. This interrupted the intended continuity of the sessions and created a three-week gap for one of the pairs. To adjust for this, the second session featured heavy Week 1 recap before introducing any new topics [Appendix 6].

It was originally intended for the two pairs of pupils to be rotated each week, with the aim of creating more variety in conceptual discussion. However, due to the timetable clash, one pair fell a week behind of the session schedule, so the pairs had to be fixed for the remaining sessions. This decision was made instead of trying to rush content, as increasing the pace of the sessions would likely have lessened conceptual security, moving the project away from its main aims. As the project sessions continued, it became evident that this pair was less motivated and less engaged with the project, slowing the pace at which they covered content. To stay in line with the project's aim of securing conceptual depth in the main operations, the decision was made to restrict their content to addition and subtraction of fractions, alongside the supporting ideas of LCM and equivalence. This decision was made to prioritise depth of content over breadth. The other pair continued with the original content of all four operations.

5. Evaluation and Legacy

5.1 Evaluation Against the Project Aims

The project was largely successful in its primary aim of leading teaching with the reasoning behind fractions methods. Through the scaffolded session cycle, pupils were asked to explain their methods with mathematical reasoning before applying them to questions for fluency. The rationale behind each method was provided by the challenge questions, which demonstrated the limits of the pupils' existing methods, creating a reason for the next concept to be introduced. Supporting topics such as equivalence and LCM were introduced to underpin these methods, helping them to stay connected and relevant to the fraction operations being taught. Tying methods to their terminology, as with "lowest", "common", and "multiple", helped to demystify them and support recall. The achievement of this aim is evidenced in Appendix 4, which details the "really nice, concept-driven approach".

There is also evidence that the project achieved its second aim of supporting pupils in distinguishing between the methods for the core fraction operations. Structuring the sessions to introduce operations sequentially helped to optimise working memory, keeping the procedures distinct. Frequent recaps kept previous content in the pupils' minds so that they could more easily see the differences between the procedures. The mixed questions exercises helped pupils develop familiarity with swapping between calculations involving different operators. The success of this was most evident in the final recap session, when pupils were successful in completing the mixed exercises. However, this aim was achieved unevenly across the two pairs, as one progressed through all four operations, while the other only covered addition and subtraction to prioritise conceptual depth.

The third aim, to encourage pupils to refine their conceptual understanding of fractions through discussion, was one of the strongest aspects of the project. Written explanations are also considered here, as they provided another way for pupils to capture their conceptual understanding in words. Appendix 4 describes having the pupils explain in their own words as a "very strong pedagogical device". Written definitions helped to reveal hidden misconceptions that numerical answers alone could not, while discussion helped to scaffold pupils towards more secure conceptual understanding [Figure 2]. For example, using the fractions wall resource to check solutions allowed pupils to explore and develop methods for themselves, being guided towards the correct method through targeted teacher prompting. The success of discussion is evidenced in feedback from [redacted] staff, which details the session leader asking pupils questions that "enable them to explain their reasoning" [Appendix 7].

The fourth aim, supporting pupils in developing greater confidence and independence in fractions, was visible in the sessions. The project supported independence through the gradual reduction of temporary teacher support. As this support decreased, pupils were encouraged to rely more heavily on the methods that they had written in their own words. This, again, was most evident in the final recap session when pupils independently completed mixed exercises, with their self-written methods as their main scaffold [Appendix 5]. Through correcting their own methods using the fractions wall and discussion, pupils began to develop the diagnostic tools for identifying misconceptions in their mathematical work, helping them take greater responsibility for their learning. An example of growing confidence came when a pupil spotted and corrected a conceptual mistake in a worksheet Figure 6. This demonstrated that she was engaging with the material confidently enough to question the teaching materials. An increase in confidence is also mentioned in the university observation form, which states that "the confidence of the children seemed to grow throughout the session" [Appendix 4].

Overall, the project was successful in meeting its four main aims within the sessions themselves, however the evidence is less strong for long-term application going forward.

5.2 Other Achievements

Beyond these four aims, a key achievement of the project was the responsive adaptation of the sessions. The spotlight pupils had various mathematical gaps and difficulties with maintaining focus, so keeping the project sessions responsive was crucial to success. Worksheets were adapted week-by-week to recap the topics that pupils struggled with, and fill-in-the-blank-style activities were added after the repeated writing of methods became too demanding. The best example of this is, again, in Figure 6, when a worksheet mistake was spotted by a pupil. This was adapted for the following session, but still provided a useful teaching moment as the mistake was discussed to give the pupils more exposure to refining mathematical definitions. This responsiveness was important because it allowed the project to remain focused on the main project aims, while still adapting to the pupils' needs.

Another achievement of the project was the prioritisation of depth over breadth when one pair of pupils progressed more slowly than expected. Despite limiting the range of operations covered, this prevented the sessions from rushing through the full planned content. Instead, addition and subtraction were consolidated more securely, alongside the supporting ideas of equivalence and LCM. This was a strength of the project as it prioritised the central focus of conceptual understanding over curriculum coverage.

5.3 Limitations

The primary limitation of the project was the restricted number of sessions, and the lack of continuity between them. The cancellation caused by the Religious Education inspection, paired with timetable clashes and the half-term break, interrupted the intended narrative flow between the sessions. This particularly affected one pair who had a three-week gap between adjacent sessions. To adjust for this, the following sessions required more recap than was originally planned, reducing the amount of available time for consolidation or past SATs questions practice.

Another limitation was the small scale of the project. Working with two pairs of spotlight pupils enabled responsive teaching which was one of the main strengths of the project. However, the lack of sample size means that any conclusions made from the project are not truly representative of pupils across the country. The small scale of the project also prevented the collection of quantitative data. Thus, this evaluation relies on pupil work and feedback, rather than any formal assessment, so it would not be appropriate to make more general conclusions about improvement in attainment.

A further limitation was that the success of the project depended heavily on responsive teaching. Although the worksheets determined the structure of each session, most of the pupils' conceptual development was a result of effective prompting, teacher-led discussion, immediate feedback and adaptation. The worksheets were shared with the staff at [redacted], however the resources alone may not completely reproduce the reasoning-led approach without the same manner of teaching.

5.4 Legacy and Future Development

The project's clearest legacy is the resources left behind for staff to deliver the sessions again with other pupils. The pupils themselves kept their final-session recap sheets to support future revision, as they contained their handwritten methods and solutions for all of the content covered. This will help with continuing the project's aim of developing pupil independence into the future.

However, the more important legacy is the reasoning-led scaffolding cycle that underpinned the worksheets, not the worksheets themselves. This model of intervention was successful in achieving the project's aims of developing conceptual understanding, independence, and confidence in fractions. Going forward, this model could be adapted to other educational applications, as the strengths themselves are beyond the specific content of the sessions. Future versions of the project resources therefore could include guidance about suggested teacher prompts and common pupil misconceptions.

If the limitations imposed by timetabling issues were not present, the project might have benefited from covering areas of the curriculum that develop understanding of the value of fractions, beyond them simply being mathematical objects to be manipulated [Appendix 7]. This would help contextualise the operations-based problems, strengthening the reasoning-led approach. Additionally, pre-project and post-project assessments would have helped to make the outcomes more measurable, allowing more general claims to be made about the project's successes.

6. Conclusion

This project was centred around using a reasoning-led approach to help Year 6 spotlight pupils develop their understanding of the core fractions operations. The strongest achievement was the use of discussion and writing to diagnose pupil misconceptions, helping them to refine their conceptual understanding. Confidence and independence were developed via scaffolding, and frequent recap and mixed questions assisted pupils in making clearer distinctions between the methods. However, the disrupted continuity and small-scale nature of the project limited both the breadth of content covered and the depth of conclusions that can be made. The dependence on responsive delivery limits the impact of the legacy that the project will leave.

Nevertheless, it was demonstrated within the project sessions that conceptual discussion, responsive adaptation, and a scaffolded approach to introducing content can help pupils with fragmented understanding of fractions to develop confidence and independence in the core operations.

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8. Appendices

Appendix 1 – example of worksheets' scaffolded cycle

<u>Adding Fractions</u>

1. independent attempt at a problem

Can you add these two fractions? $\frac{2}{4} + \frac{1}{4} = \boxed{}$

3. teacher demonstrates solution

Can you explain what method you used to do this?

2. pupils explain method

4. pupils correct/expand upon written method

Try these 3 practice questions!

1. $\frac{1}{8} + \frac{6}{8} = \boxed{}$
2. $\frac{7}{12} + \frac{5}{12} = \boxed{}$
3. Challenge! $\frac{1}{2} + \frac{1}{4} = \boxed{}$

5. practice questions for fluency

6. challenge question, intended to break the method

What was different about question 3?

7. pupils identify why the method was unsucessful

<u>Equivalent Fractions</u>

8. challenge question informs next discussion

Let's look at our fractions wall!

- Are these two fractions ***equivalent***? $\frac{1}{2}$ and $\frac{3}{6}$
- How about these two? $\frac{5}{6}$ and $\frac{7}{12}$

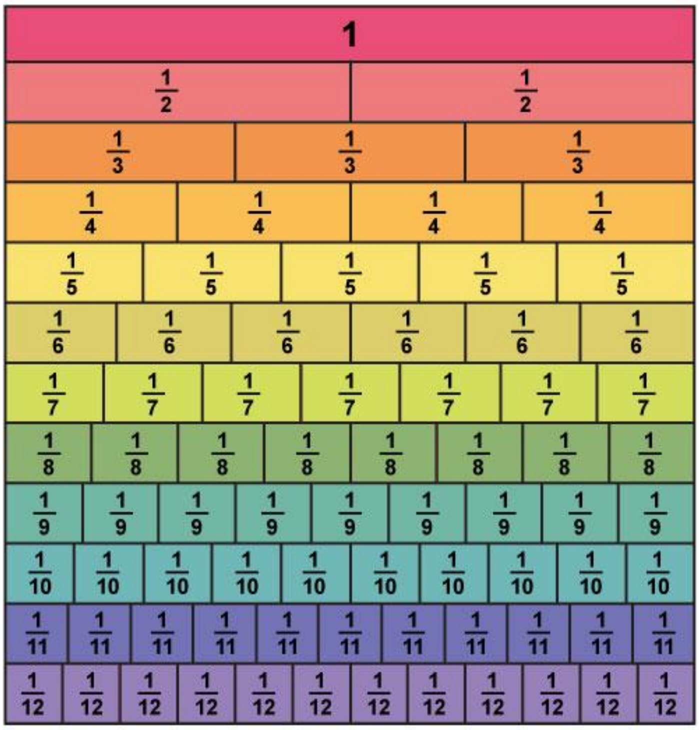
Appendix 2 – multiplication grid from [TimesTablesKids.com]

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

TimesTableKids.com

Appendix 3 – fractions wall from Brother Creative Center

BROTHER CREATIVECENTER



Appendix 4 – Durham University Placement Observation Record

Durham University

Mathematical Sciences in Schools
[MATH3481]
Level 3 Placement Observation Record

Name (student being observed) Louis Durston-Wyatt

Name (host teacher) [redacted]

Name (observer) [redacted]

Title / type of session In-lesson-time small group Project session

Date 4/3/26 **Location** [redacted] **Period** 12:30 → 13:45

Preliminary information (as available):
Small group intervention sessions. 2 x 2 children, working on rationals Spends time across [redacted] Yr 4 and [redacted] Yr 6 classes.

Time:	**Observation record:**
12:30 Arrival	Chat with Head of Maths, [redacted]. Louis had spent time in [redacted] and [redacted] classes. For project, takes out kids from [redacted]'s Yr 6.
	RM extremely complementary about Louis' contribution to school in general, and project.
13:00	Project session started, with Louis taking 2 children out of class. Louis started with a recap about last time, with an example to check memory. Louis has a fantastic manner with the children, showing very good encouragement. An "Oh" of realisation is great evidence of success..
13:05	Started via investigation, asked them to try and define equivalent fractions. Let both students explain in writing and words their thoughts – this seemed a very strong pedogogical device, before moving on to examples

(continue over if required)

13:15: Started new topic (add, subt) linking it to previous work. Really nice concept-driven approach.

(contd.)

- Throughout there was an excellent mix of asking students to describe thoughts, and answer question. Rather than repetition.
- Depending on aims of project, one could try to get one student to explain to the other (if emphasis is on being able to communicate ideas)
- Level of worksheets seemed excellent as students did not get answers too quickly, and sometimes needed hints. Louis was excellent at getting students to correct wrong answers by working through.
- The worksheets looked v good, but maybe one could make some bits more varied? The chat does this to a great extent
- Very good intro to LCM (excellent linking of terminology to meaning using a supplementary tables sheet). The confidence of the children seemed to grow throughout. It was v impressive to keep students so engaged in lesson-like environment.

13:55 – Very good appreciation of (and disarming intro to) multiplication of fractions, especially anticipation that it would be easier than +/−. Excellent extension of end.

Overview and comments:

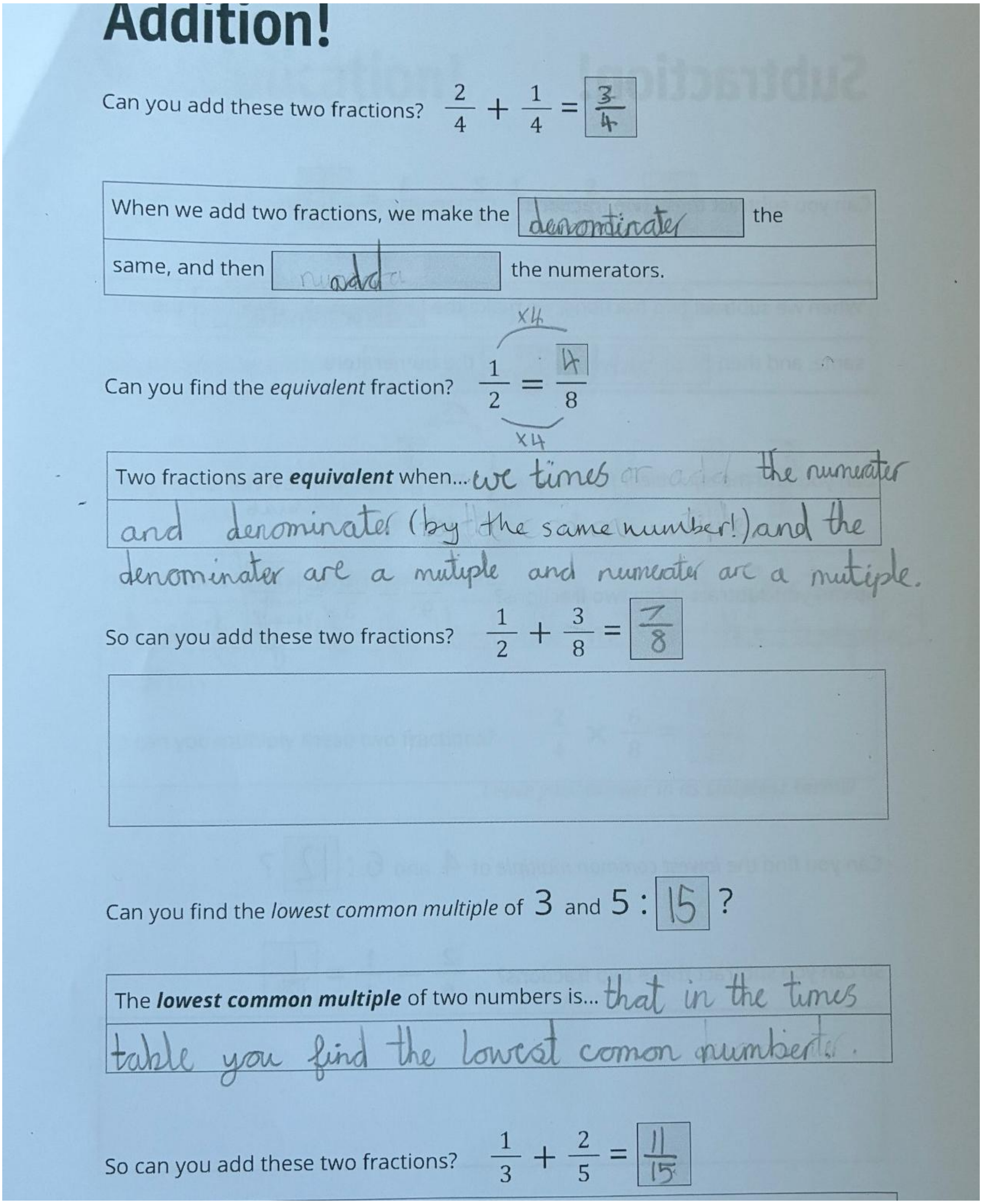
- Louis has plenty of opportunity to both observe classes and lead small group/one-on-one sessions.
- Placement seems very successful with teacher very complementary, both in terms of general help and value of project.
- Project location is very suitable (though if class door is fully closed, it is a little unsupervised despite glass window.) Length is variable/controlled by Louis, and replaces non-Maths/English lesson.
- Project is slightly constrained by limiting factors out of Louis' control (# of students/ scope of material), but Louis seems to have done an excellent job of maximising the benefits, with well structured concept-driven design with good difficulty gradient and emphasis on explanation and the description of ideas.
- Louis has an outstanding manner with the children and gave some extremely useful and lucid explanations. In fact, the quality of the session really hung on Louis clear ability in one-on-one teaching. Louis may want to think about how to best articulate or evidence the good practice on show for pres/report.

Signed (student being observed) ……………………………………………………………

Signed (observer) [redacted]……………………………………………………………

Asides:
- How much do the children appreciate the value of fractions, or do they just see them as objects they need to learn to manipulate? The aim may well be to ensure they can do the latter.
- What impact does taking kids out of a, say Geography, lesson have? What benefits does teacher see?

Appendix 5 – pupil example of final session worksheet (without mixed problems)



Addition!

Can you add these two fractions? $\frac{2}{4} + \frac{1}{4} = \boxed{\frac{3}{4}}$

When we add two fractions, we make the denomtinater the same, and then add the numerators.

Can you find the *equivalent* fraction? $\frac{1}{2} \overset{\times 4}{=} \frac{\boxed{4}}{8}$ ($\times 4$)

Two fractions are **equivalent** when... we times ~~or add~~ the numerater and denominater (by the same number!) and the denominater are a mutiple and numerater are a mutiple.

So can you add these two fractions? $\frac{1}{2} + \frac{3}{8} = \boxed{\frac{7}{8}}$

Can you find the *lowest common multiple* of 3 and 5 : $\boxed{15}$?

The ***lowest common multiple*** of two numbers is... that in the times table you find the lowest comon number.

So can you add these two fractions? $\frac{1}{3} + \frac{2}{5} = \boxed{\frac{11}{15}}$

Subtraction!

Can you subtract these two fractions? $\frac{2}{4} - \frac{1}{4} = \boxed{\frac{1}{4}}$

When we subtract two fractions, we make the	denominator	the
same, and then	Subtract	the numerators.

Can you find the *equivalent* fraction? $\frac{1}{3} \overset{\times 3}{=} \frac{\boxed{3}}{9}$ ×3

So can you subtract these two fractions? $\frac{7}{9} - \frac{1}{3} = \boxed{\frac{4}{9}}$

Can you find the *lowest common multiple* of 4 and 6 : $\boxed{12}$?

So can you subtract these two fractions? $\frac{2}{6} - \frac{1}{4} = \boxed{\frac{1}{12}}$

Multiplication!

Can you multiply these two fractions? $\frac{1}{3} \times \frac{2}{5} = \boxed{\frac{2}{15}}$

When we multiply two fractions, we [times] the	
numerators, multiply the [denominater] and simplify.	

Can you put this fraction in its *simplest terms*? $\frac{12}{32} = \frac{\boxed{6}}{\boxed{16}} = \frac{\boxed{3}}{\boxed{8}}$

or keep dividing it by any number until it can't be divided anymore!

A fraction is in its ***simplest terms*** when... you get a fraction then half it so it will be in the simplest form.

So can you multiply these two fractions? $\frac{2}{4} \times \frac{6}{8} = \boxed{\frac{3}{8}}$

*Leave your answer in its **simplest terms**!*

Fractions of a Whole Number!

*Leave all your answers in their **simplest terms**!*

Can you fill in the blanks to find the answer to this question?

$$\frac{5}{6} \times 120 = \frac{\boxed{600}}{\boxed{6}} = \boxed{100}$$

When we ***multiply*** a fraction by a whole number, we	Multiply
the numerrater by the number and simplify.	

Try another using this method!

$$72 \times \frac{3}{8} = \boxed{27}$$

027
8|216

72
× 3
216

What word can we replace '×' with? of

So what is $\frac{2}{3}$ of $5 = \boxed{\frac{10}{3}}$

$2 \times 5 = 10 = \frac{10}{3}$ 10

3|100

Dividing Fractions!

*Leave all your answers in their **simplest terms**!*

Let's try this: $\frac{10}{3} \div 5 = \frac{\boxed{10}}{\boxed{3 \times 5}} = \boxed{\frac{10}{15}} = \frac{2}{3}$

When we ***divide*** a fraction by a whole number, we	times
the denominatater by the number and simplify.	

Try some questions!

1. $\frac{8}{9} \div 4 = \boxed{\frac{2}{9}}$

$\frac{8}{9} \div 4 \quad \frac{8}{9 \times 4} = \frac{8}{36} = \frac{2}{9}$

2. $\frac{3}{7} \div 5 = \boxed{}$

Appendix 6 – adding fractions recap from beginning of session 2

Adding Fractions: Recap!

$\frac{1}{3} = \frac{2}{6}$

Adding Fractions

When we add fractions, what do we do to the ***numerator*** and the ***denominator***?

Make the denominators equal and then add
the numerators.

So what is $\frac{4}{6} + \frac{1}{6} = \boxed{\frac{5}{6}}$?

$\frac{1}{3} = \frac{2}{6}$

Equivalent Fractions

Two fractions are ***equivalent*** when... the numerators are
multiples and so are the denominators.

Fill in the gap: $\overbrace{\frac{2}{5} = \frac{\boxed{4}}{10}}^{\times 2}$ ($\times 2$)

$= \frac{4}{10}$

$\frac{2}{5} + \frac{1}{10}$

So what is $\frac{2}{5} + \frac{1}{10} = \boxed{\frac{5}{10}}$?

$\frac{2}{5} \overset{\times 2}{=} \frac{4}{10}$ ($\times 2$) $\quad \frac{4}{10} + \frac{1}{10} = \frac{5}{10}$

Lowest Common Multiple

The ***lowest common multiple*** of two numbers is...

The lowest common multiple of 9 and 6 is: $\boxed{18}$

***(Hint**: you can use the multiplication table!)*

So what is $\frac{2}{9} + \frac{1}{6} = \boxed{\frac{7}{18}}$?

$$\frac{2}{9} = \frac{4}{18} \quad \frac{1}{6} = \frac{3}{18} \quad \frac{4}{18} + \frac{3}{18} = \frac{7}{18}$$

Appendix 7 – feedback from [redacted] staff on the communication skills of the mathematics undergrad (session teacher)

3a) The undergraduate's aptitude and potential as a science communicator (25%) (including oral & written communication skills, presentation skills, delivery at an appropriate level, enthusiasm for the subject, rapport with children, use of questions and responses to questioning, adherence to syllabus & learning aims & outcomes):

	Mark:	
On this aspect the student was Exemplary (86% - 100%)	25	Put your mark out of 25 here!

3b) Comments:

Louis has develeoped an execellent relationship with the children he has been working with. He is organised and very competent. He asks appropriate questions to staff and also to children to enabl them to expalin their reasoning. He has been able to focus on particular strands of the curriculum and understand their relevance within the curriculum.